

READING THE SIDE  
OF A SQUARE

AMS CONFERENCE  
PRESENTATION  
ON  
SQUARING AND SQUARE ROOT

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## THE LAST PASSAGES OF THE BINOMIAL AND TRINOMIAL

Before this work is given, the child must be very familiar with the squares divided into a binomial and a trinomial, and must know how to multiply a binomial by a binomial and a trinomial by a trinomial.

### The Product of a Binomial with the Golden Beads

Material:

- ⇒ The golden bead material
- ⇒ A large box of bead bars 1 to 10

Presentation:

We are going to multiply binomials that do not give a square for a product.

1. Place the equation on the table, making the multiplier in cards and the multiplicand with bead bars:

$$( \text{OOOOOOO} + \text{OOO} ) \times ( 4 + 2 )$$

2. This means that we must multiply 7 + 3 by 4 first, and then 7 + 3 by 2.
3. Place four sevens and four threes on the table.
4. Then, turn over the card 4.
5. Next, place two sevens and two threes.
6. Then push these two sets of bead bars together.
7. When we put the bead bars together, the figure we have is not a square; it is a rectangle.

$( \text{OOOOOOO} + \text{OOO} ) \times ( 4 + 2 )$

OOOOOOO	OOO	
OOOOOOO	OOO	
OOOOOOO	OOO	X 4
OOOOOOO	OOO	
OOOOOOO	OOO	
OOOOOOO	OOO	X 2

Note: This is an introduction with the material; it is not a mathematical representation. The parentheses are used differently here. Later, they will be used in the usual way.

Next, show the children how the process is written mathematically:

$$\begin{aligned}
 (7 + 3) \times (4 + 2) &= \\
 (7 + 3) \times 4 &= (7 \times 4) + (3 \times 4) = 28 + 12 = 40 \\
 + (7 + 3) \times 2 &= (7 \times 2) + (3 \times 2) = 14 + 6 = 20 \\
 &= \underline{42 + 18 = 60}
 \end{aligned}$$

When this example is completed, do the inverse:

$$( \text{OOOO} + \text{OO} ) \times ( 7 + 3 )$$

The result will be the same mathematically, but the figure will be different.

$$\begin{aligned}
 (4 + 2) \times (7 + 3) &= \\
 (4 + 2) \times 7 &= (4 \times 7) + (2 \times 7) = 28 + 14 = 42 \\
 + (4 + 2) \times 3 &= (4 \times 3) + (2 \times 3) = 12 + 6 = 18 \\
 &= \underline{40 + 20 = 60}
 \end{aligned}$$

$( \text{OOOO} + \text{OO} ) \times ( 7 + 3 )$

OOOO	OO	
OOOO	OO	
OOOO	OO	x 7
OOOO	OO	
OOOO	OO	
OOOO	OO	
OOOO	OO	
OOOO	OO	x 3
OOOO	OO	

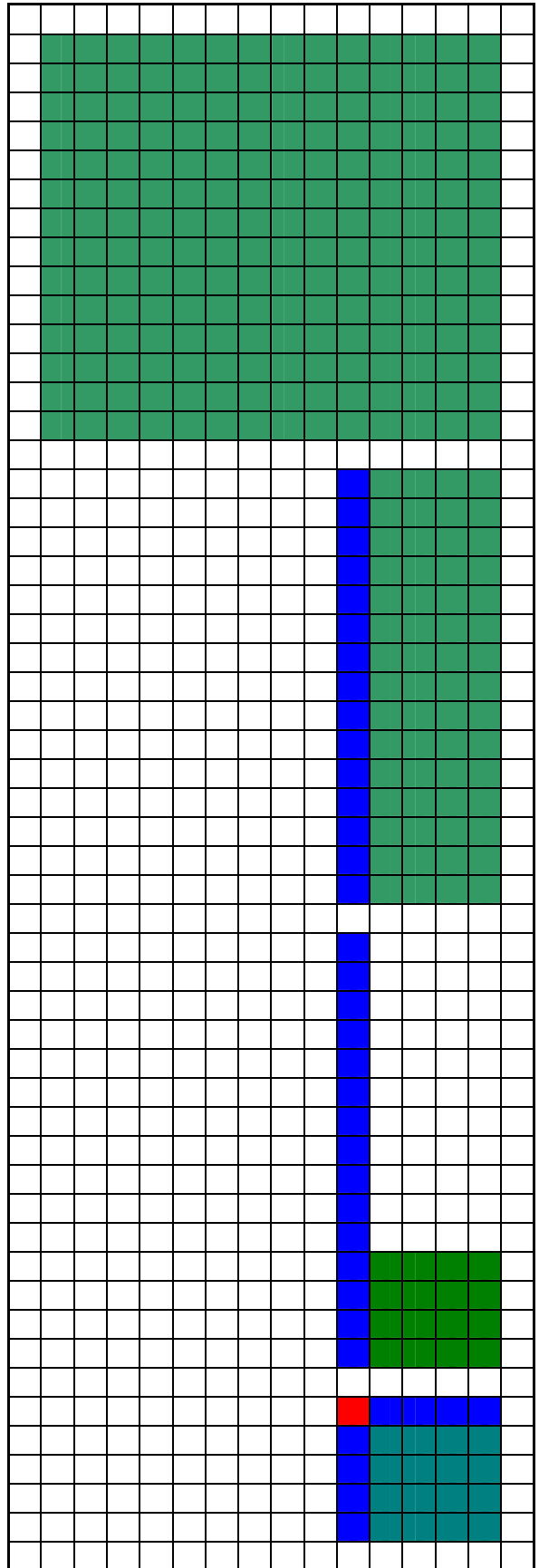
## Passage from a Real Square to a Symbolic Square

### Materials:

- ⇒ the pegboard and pegs
- ⇒
- ⇒ two or more stamp games
- ⇒ Golden beads

### Presentation:

1. We have made the square of 14 with the golden bead material.
2. Now, we are going to make the square of 14 with this material.
3. We are going to form a square with 14 pegs for a side.
4. Ask the children to do this - placing the sides of the square first, then filling in the center, and using the green pegs.
5. This is an introductory step, in which the child sees that  $14 \times 14$  is 196. He has that number in green pegs.
6. After the square of green pegs has been formed, tell the children that you want to form another square. That is, you want to transform this square into another square of the same value, only much smaller.
7. Take a green peg from the box and ask the children what its value is. One.
8. Take a blue peg and ask what its value is. Ten.
9. Then, take a red peg and ask its value. One hundred.
10. Now, recall that the requirements for the new square are that it must be of the same value as this large square of green pegs, and it must be smaller in size. To make the new square, you are going to use pegs of different values, whereas in the green square, all the pegs had the value of one.
11. Begin at the top left. Count ten green pegs to the right.
12. Remove them and replace them with a blue peg.
13. Proceed in the same way for all the rows.
14. Now, there is a column of blue pegs, and to the right, four columns of green pegs.
15. Begin with the first of these green columns. Count down ten green pegs and replace them with a ten.



16. Do the same with the other columns.
17. Now you have the very interesting figure seen in the illustration.
18. Count down the column of blue pegs and replace the first ten with a red peg for 100.
19. Now the figure is a square, made up of one hundred, 8 tens and 16 units - which equal 196.
20. Count the pegs to verify that there are 196.
21. The child can compare this symbolic square with the one in golden beads, which should be left until this activity is complete.
22. If the notation was not done with the beads, it is done now.  
(The real square can also be made with the green stamp and transformed to symbolic)

## **Second Exercise:** Symbolic Square

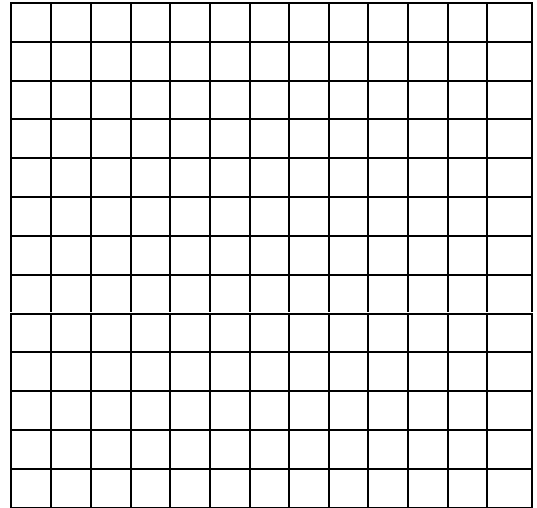
**Materials:** Peg board and pegs

**Presentation:**

1. In the second exercise, the square is not made first with green pegs. This was done as an introductory presentation.
2. The notation is done, and then the square is constructed on the peg-board.
3. Example:  $25^2$

Notation:

$$\begin{aligned} 25^2 &= (20 + 5)^2 = (20 + 5) \times (20 + 5) = \\ &(20 \times 20) + (20 \times 5) + (5 \times 20) + (5 \times 5) = \\ 20^2 &+ 2(20 \times 5) + 5^2 = \\ 400 &+ 200 + 25 = \\ 625 & \end{aligned}$$

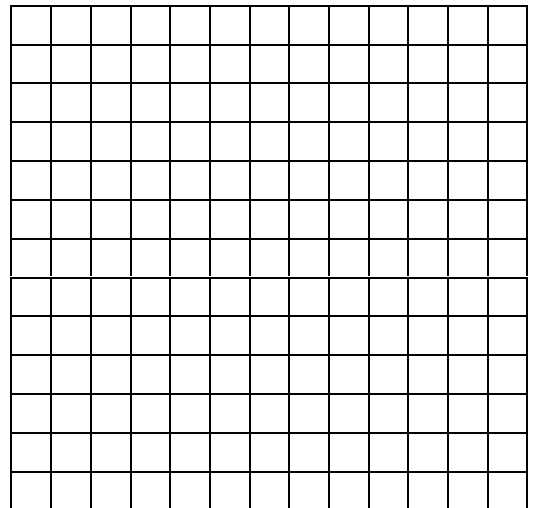


4. The pegs in the hierarchical colors are placed on the pegboard in the square for 25.

**Another Example:**  $32^2$

Notation:

$$\begin{aligned} 32^2 &= (30 + 2)^2 = (30 + 2) \times (30 + 2) = \\ &(30 \times 30) + (30 \times 2) + (2 \times 30) + (2 \times 2) = \\ 30^2 &+ 2(30 \times 2) + 2^2 = \\ 900 &+ 2(60) + 4 = \\ 900 &+ 120 + 4 = 1024 \end{aligned}$$



The pegs in the hierarchical colors are placed on the pegboard in the square for 32.

**Direct Aim:** 1. Understanding why in the squares of numbers we proceed hundred by hundred.

2. To make the child conscious that it is important to carry out large multiplications with non-hierarchical material to show him how to reduce a real square to a hierarchical square.
3. The geometrical representation of multiplication in the hierarchical colors.

**Indirect Aim:** The preparation for square root

# Binomial Guide Chart


# Trinomial Guide Chart


n	$n^2$	n	$n^2$	n	$n^2$
1	1	10	100	100	10,000
2	4	20	400	200	40,000
3	9	30	900	300	90,000
4	16	40	1600	400	160,000
5	25	50	2500	500	250,000
6	36	60	3600	600	360,000
7	49	70	4900	700	490,000
8	64	80	6400	800	640,000
9	81	90	8100	900	810,000



## Second Level: Square Root with the Pegboard

In order to make the work clearer for the child; it is good to make the notation in the hierarchical colors, for the beginning example with the pegboard.

### Presentation:

1. Write the notation:  $\sqrt{625}$
2. Place the Quantity in pegs in the bowls:
3. As with the golden beads what square we can make with 6 hundreds.
4. Take the 6 red pegs ... What square can we form?
5. Place the pegs in this position:
6. We have made a square with side 2 - therefore, the first digit in the root, the tens digit, is 2.
7. Write 2 in the notation.
8. How many pegs did we use? 4.
9. Write 4 and subtract to see how many hundreds were not used. 2
10. Exchange the two hundreds for tens.
11. Now there are 22 pegs in the tens bowl.
12. In the notation, bring down the 2 for the original two tens.
13. Begin forming rectangles on the sides of the square, placing two blue pegs on one side, then the other and alternating.
14. It is possible to make two rectangles with a length of 5.
15. Record the 5 in pencil in the notation; because we are not sure it is the number we want.
16. How many tens did we use? 20.
17. Write this in the notation and subtract.
18. Change the two remaining tens for units.
19. Bring down the last digit. We now have 25 units.
20. Try to fill in the space at the bottom right with these pegs.
21. How many pegs did I use? 25.
22. Record this in the notation and subtract.
23. There is no remainder because 625 is a perfect square.
24. At this point, introduce the guide chart. It is to show that we start with hundreds, then tens, then units.



$$\begin{array}{r} \sqrt{625} \quad | \quad 2 \\ -4 \phantom{0} \\ \hline 22 \end{array}$$



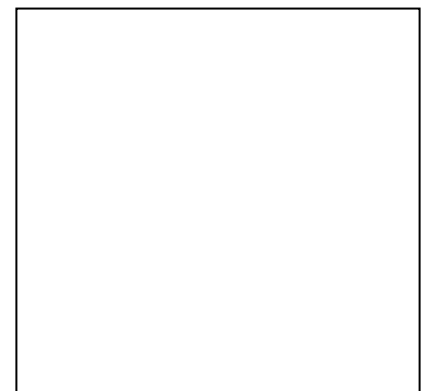
$$\begin{array}{r} \sqrt{625} \quad | \quad 25 \\ -4 \phantom{0} \\ \hline 22 \\ -20 \\ \hline 25 \\ -25 \\ \hline 0 \end{array}$$

**Note:** In carrying out the square root of a Binomial, there are three parts:

1. Research and confirmation of the tens digit  
(Steps 4 – 9 above)
2. Research of the units digit  
(Steps 10 – 17 above)
3. Confirmation of the units digit  
(Steps 18 – 23 above)

The first and second parts are called the “alive” parts, because in these processes we find the square root.

The third is called the “dead” part, because we are merely confirming what we have already discovered. We do this by filling in the last part of the square.



## Writing the Root: The Root of a Trinomial

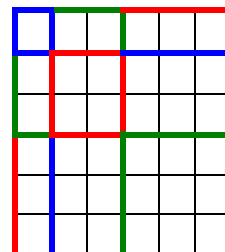
Ref. Illustrative Psycho Arithmetic, pp. 311-316

### Materials:

- ◆ Guide Chart
- ◆ Pegboard and pegs

### Presentation:

1. Show the guide chart \_\_\_\_\_
2. Write the problem  $\sqrt{55696}$
3. Place the pegs in the bowls. These represent the radicand.
4. Mark off the digits.
5. There will be three digits in the root, H, T, and U

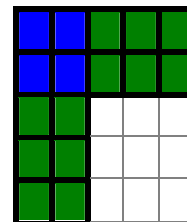


$$\begin{array}{r} \sqrt{55696} \\ \hline \text{H T U} \end{array}$$

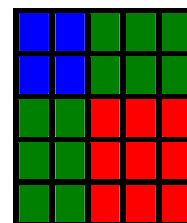
- 1) Research the first digit of the root.  
What square can be formed with these 5 ten thousands?  
The square of 2  
Therefore, 2 is the first digit of the root (the hundreds digit).  
We use 4 ten thousands; Write 4 and subtract.  
Now, exchange the remaining ten thousand for thousands.  
We will be working with 15 thousands.



- 2) Second passage - Looking for the second digit of the root  
Place the thousand pegs. It is possible to place 3 sets of thousands on each side of the square. Therefore we predict the ten digit of the root is 3.  
Write the tens digit in pencil in the tens place in the root.

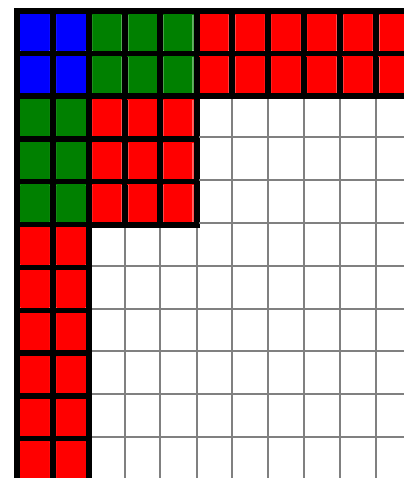


- 3) This phase will give confirmation to the second digit of the root – we must have enough hundreds to construct a square.  
Exchange the remaining 3 thousands for 30 hundreds.  
Then verify the second digit of the root, i.e., construct a square of hundreds (3 x 3).  
Nine pegs are needed.  
Subtract nine.

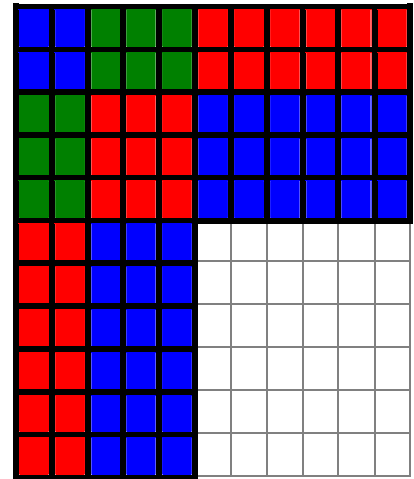


- 4) Construct rectangles with the remaining hundreds, extending both sides as far as possible.  
Each rectangle is 6 twos.  
Make the notation.

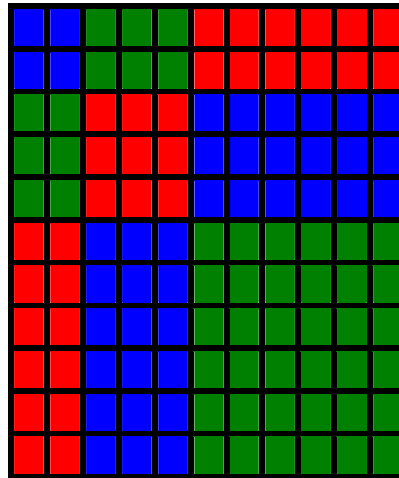
The red line indicates that the “alive” part is finished.



- 5) Confirmation of the third digit:  
 Exchange the remaining hundreds for tens.  
 Extend both sides of the second square: 6 sets of 30.  
 These rectangles support the rectangles of hundreds.  
 Pegs used: 36. Write 36 and subtract.

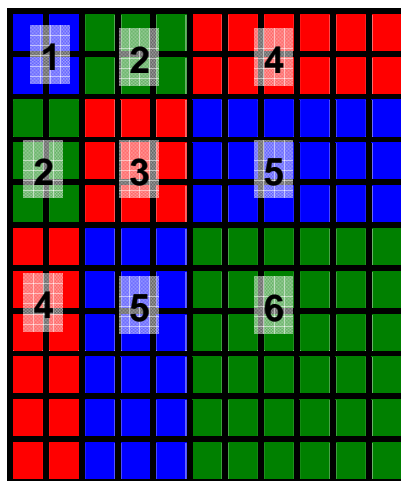


- 6) Second confirmation of the third digit.  
 Exchange the remaining tens.  
 Fill in the final square:  $6^2$ .  
 Pegs used: 36.  
 Record.

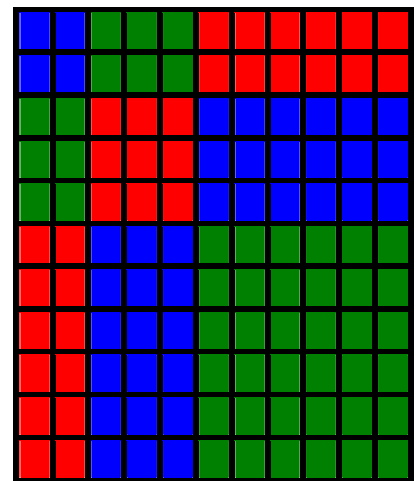


Analyze the square.

Label with slips the 6 moments.



Divide the dead from the live part. Use small black strips of paper.



### First proof

Analyze each figure -its placement, dimensions, how the hierarchy was formed and value.

1. Ten thousands	$2^2$	=	H x H	=	200 x 200	=	40,000
2. Thousands	$2 \times 3$	=	H x T	=	200 x 30	=	6,000
	$3 \times 2$	=	T x H	=	30 x 200	=	6,000
3. Hundreds	$3^2$	=	T x T	=	30 x 30	=	900
	$2 \times 6$	=	H x U	=	200 x 6	=	1,200
	$6 \times 2$	=	U x H	=	6 x 200	=	1,200
4. Tens	$3 \times 6$	=	T x U	=	30 x 6	=	180
	$6 \times 3$	=	U x T	=	6 x 30	=	180
5. Units	$6^2$	=	U x U	=	6 x 6	=	36
							<u>55,696</u>

Do second proof. Multiply the root by itself.  $236 \times 236 =$  \_\_\_\_\_

Activity: Make a colored paper design of the square.